Examiners' Report: Final Honour School of Mathematics Part B Trinity Term 2020

March 5, 2021

Part I

A. STATISTICS

- **Numbers and percentages in each class.**

  See Table 1.

	Numbers					Percentages %				
	2020	(2019)	(2018)	(2017)	(2016)	2020	(2019)	(2018)	(2017)	(2016)
I	73	(59)	(58)	(51)	(56)	46.5	(39.07)	(38.16)	(38.63)	(39.72)
II.1	66	(67)	(67)	(64)	(58)	42.04	(44.37)	(44.08)	(48.48)	(41.13)
II.2	13	(20)	(25)	(11)	(24)	8.28	(13.25)	(16.45)	(8.33)	(17.02)
III	4	(4)	(2)	(3)	(3)	2.55	(2.65)	(1.32)	(2.27)	(2.13)
P	1	(0)	(0)	(2)	(0)	0.64	(0)	(0)	(1.52)	(0)
F	0	(1)	(0)	(0)	(0)	0	(0.66)	(0)	(0)	(0)
Total	157	(151)	(152)	(132)	(141)	100	(100)	(100)	(100)	(100)

Table 1: Numbers and percentages in each class

- **Numbers of vivas and effects of vivas on classes of result.**

  As in previous years there were no vivas conducted for the FHS of Mathematics Part B.

- **Marking of scripts.**

  BE Extended Essays, BSP projects, and coursework submitted for the History of Mathematics course, the Mathematics Education course and the Undergraduate Ambassadors Scheme, were double marked.

  The remaining scripts were all single marked according to a pre-agreed marking scheme which was strictly adhered to. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**

  See Table 4 on page 11.

B. New examining methods and procedure in the 2020 examinations

In light of Covid 19, the department took steps to mitigate the impact of the pandemic on academic performance. This included changing the examinations to open-book version of the standard exam papers, reducing the units required from 8 to 6, the introduction of the safety net and Declared to have Deserved Honours. In addition, the method of assessing mitigating circumstances at the exam board was changed. An additional hour was also added on to the Mathematics exam duration to allow candidates the technical time to download and submit their examination papers via Weblearn. Given the unusual circumstances and impact of Covid-19, ranking was only used for the purposes of awarding prizes. The introduction of the safety net (which was applied to cohorts) meant that the overall average and hence rank was not well defined.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

Due to the uncertainty with the pandemic, the department decided that exams will be taken online for Trinity Term 2021.

D. Notice of examination conventions for candidates

The first Notice to Candidates was issued on 24 February 2020 and the second notice on 5 May 2020.

All notices and the examination conventions for 2020 are on-line at http://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments.

Part II

A. General Comments on the Examination

The examiners would like to convey their grateful thanks for their help and cooperation to all those who assisted with this year's examination, either as assessors or in an administrative capacity. The chairman would particularly like to thank Elle Styler for administering the whole process with efficiency, and also to thank Nicole Collins, Charlotte Turner-Smith and Waldemar Schlackow.

In addition the internal examiners would like to express their gratitude to Professor Marco Schlichting and Professor Michal Branicki for carrying out their duties as external examiners in a constructive and supportive way during the year, and for their valuable input at the final examiners' meetings.

Standard of performance

The standard of performance was broadly in line with recent years. In setting the USMs, we took note of

- the Examiners' Report on the 2019 Part B examination, and in particular recommendations made by last year's examiners, and the Examiners' Report on the 2019 Part A examination, in which the 2020 Part B cohort were awarded their USMs for Part A;
- a document issued by the Mathematics Teaching Committee giving broad guidelines on the proportion of candidates that might be expected in each class, based on the class percentages over the last five

years in Mathematics Part B, Mathematics & Statistics Part B, and across the MPLS Division.

Having said this, as in Table 1 the proportion of first class degrees in Mathematics alone awarded (39.07%) was high, and the proportion of II.2 and below degrees in Mathematics awarded (13.25%) was low, compared to the guidelines. One reason for this is that the examiners consider candidates in Mathematics and in Mathematics and Statistics together when determining USMs, and this year the Mathematics and Statistics candidates performed poorly compared to the Mathematics candidates, so that the averages for the two schools combined (27.87% firsts, and 12.57% II.2 and below) are consistent with the Teaching Committee guidelines.

Setting and checking of papers and marks processing

Requests to course lecturers to act as assessors, and to act as checkers of the questions of fellow lecturers, were sent out early in Michaelmas Term, with instructions and guidance on the setting and checking process, including a web link to the Examination Conventions. The questions were initially set by the course lecturer, in almost all cases with the lecturer of another course involved as checkers before the first drafts of the questions were presented to the examiners. Most assessors acted properly, but a few failed to meet the stipulated deadlines (mainly for Michaelmas Term courses) and/or to follow carefully the instructions provided.

The internal examiners met at the beginning of Hilary Term to consider those draft papers on Michaelmas Term courses which had been submitted in time; consideration of the remaining papers had to be deferred. Where necessary, corrections and any proposed changes were agreed with the setters. The revised draft papers were then sent to the external examiners. Feedback from external examiners was given to examiners and to the relevant assessor for response. The internal examiners at their meeting in mid Hilary Term considered the external examiners' comments and the assessor responses, making further changes as necessary before finalising the questions. The process was repeated for the Hilary Term courses, but necessarily with a much tighter schedule.

Due to the Pandemic, Exam Papers were revised and set to be open book. Camera ready copy of each paper was signed off by the assessor, and then submitted to the Examination Schools.

Candidates accessed and downloaded their exam papers via the Weblearn system at the designated exam time. Exam responses were uploaded to Weblearn and made available to the Exam Board Administrator 48 hours after the exam paper had finished.

The process for Marking, marks processing and checking was adjusted accordingly to fit in with the online exam responses. Assessors had a short time period to return the marks on the mark sheets provided. A check-sum was also carried out to ensure that marks entered into the database were correctly read and transposed from the mark sheets.

All scripts and completed mark sheets were returned, if not by the agreed due dates, then at least in time for the script-checking process.

A team of graduate checkers, under the supervision of Nicole Collins, Elle Styler, sorted all the marked scripts for each paper of this examination, carefully cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the mark scheme, noting correct addition. In this way a number of errors were corrected, each change was signed by one of the examiners who were present throughout the process. A check-sum is also carried out to ensure that marks entered into the database are correctly read and transposed from the marks sheets.

Throughout the examination process, candidates are treated anonymously, identified only by a randomly-assigned candidate number, until after all decisions on USMs, degree classes, mitigating circumstances notices to examiners, prizes, and so on, have been finalized.

Standard and style of papers

It was noted at the Final Exam Board meeting that the papers 3.4 Algebraic Number Theory and 4.3 Distribution Theory were set too easy this year. These papers will need to be reviewed, especially if the exams are held as open-book again.

Timetable

Examinations began on Tuesday 2nd June and finished on Thursday 2nd July.

Determination of University Standardised Marks

We followed the Department's established practice in determining the University standardised marks (USMs) reported to candidates. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration. Calibration uses data on the Part A performances of candidates in Mathematics and Mathematics & Statistics (Mathematics & Computer Science and Mathematics & Philosophy students are excluded at this stage). Working with the data for this population, numbers N_1, N_2 and N_3 are first computed for each paper: N_1, N_2 and N_3 are, respectively, the number of candidates taking the paper who achieved in Part A average USMs in the ranges $[69.5, 100]$, $[59.5, 69.5)$ and $[0, 59.5)$, respectively.

The algorithm converts raw marks to USMs for each paper separately. For each paper, the algorithm sets up a map $R \to U$ (R = raw, U = USM) which is piecewise linear. The graph of the map consists of four line segments: by default these join the points $(100, 100)$, $P_1 = (C_1, 72)$, $P_2 = (C_2, 57)$, $P_3 = (C_3, 37)$, and $(0, 0)$. The values of C_1 and C_2 are set by the requirement that the number of I and II.1 candidates in Part A, as given by N_1 and N_2, is the same as the I and II.1 number of USMs achieved on the paper. The value of C_3 is set by the requirement that P_2P_3 continued would intersect the U axis at $U_0 = 10$. Here the default choice of *corners* is given by U-values of 72, 57 and 37 to avoid distorting nonlinearity at the class borderlines.

The results of the algorithm with the default settings of the parameters provide the starting point for the determination of USMs, and the Examiners may then adjust them to take account of consultations with assessors (see above) and their own judgement. The examiners have scope to make changes, either globally by changing certain parameters, or on individual papers usually by adjusting the position of the corner points P_1, P_2, P_3 by hand, so as to alter the map raw $\to$ USM, to remedy any perceived unfairness introduced by the algorithm. They also have the option to introduce additional corners. For a well-set paper taken by a large number of candidates, the algorithm yields a piecewise linear map which is fairly close to linear, usually with somewhat steeper first and last segments. If the paper is too easy or too difficult, or is taken by only a few candidates, then the algorithm can yield anomalous results—very steep first or last sections, for instance, so that a small difference in raw mark can lead to a relatively large difference in USMs. For papers with small numbers of candidates, moderation may be carried out by hand rather than by applying the algorithm.

Following customary practice, a preliminary, non-plenary, meeting of examiners was held two days ahead of the plenary examiners' meeting to assess the results produced by the algorithm alongside the reports from assessors. The examiners reviewed each papers and report, considered whether open book examination process affected candidates and reviewed last year's stats. The examiners discussed the preliminary scaling maps and the preliminary class percentage figures. Adjustments were made to the default settings as appropriate, paying particular attention to borderlines and to raw marks which were either very high or very low. In response to the coronavirus pandemic, a safety net was applied for certain candidates. The safety net looked at the two average USMs and the classification is based on whichever is higher. These revised USM maps provided the starting point for a review of the scalings,paper by paper, by the full board of examiners jointly with Mathematics & Statistics examiners.

Table 2 on page 9 gives the final positions of the corners of the piecewise linear maps used to determine USMs.

In accordance with the agreement between the Mathematics Department and the Computer Science Department, the final USM maps were passed to the examiners in Mathematics & Computer Science. USM marks for Mathematics papers of candidates in Mathematics & Philosophy were calculated using the same final maps and passed to the examiners for that School.

Comments on use of Part A marks to set scaling boundaries

None.

Mitigating Circumstance Notice to Examiners

In light of Covid 19, there was no separate panel meeting to discuss the individual notices to examiners. Even though the Mitigating Circumstances were initially reviewed at the preliminary meeting, all decisions on the outcome of these notices were decided at the final meeting alongside any cohort-wide decisions and the safety net being applied.

The full board of examiners considered 36 notices in the final meeting. The Board also received a total number of 13 MCEs carried over from the 2019 Part A Final Mathematics Exam Board. The examiners considered each application alongside the candidate's marks and the recommendations proposed by the Part A 2019 Exam board. The outcomes for these have been recorded on a spreadsheet report on Mitigating Circumstances Notice to Examiners from Part A. All candidates with certain conditions (such as dyslexia, dyspraxia, etc.) were given special consideration in the conditions and/or time allowed for their papers, as agreed by the Proctors. Each such paper was clearly labelled to assist the assessors and examiners in awarding fair marks.

Table 2: Position of corners of the piecewise linear maps

Paper	P_1	P_2	P_3	Additional Corners	N_1	N_2	N_3
B1.1	17.98;37	34;57	44.8;72	50;100	9	13	4
B1.2	12.29;37	24;57	39.4;72	50;100	16	17	7
B2.1	10.63;37	26;57	41;72	50;100	20	12	1
B2.2	15.11;37	26.3;57	41;72	50;100	14	8	2
B3.1	9.77;37	17;57	32;72	50;100	25	20	6
B3.2	17.23;37	30;57	40;72	50;100	4	4	1
B3.3	8.79;37	27;57	40;70	50;100	4	5	0
B3.4	21.77;37	37.9;57	50;100		21	13	7
B3.5	10.8;37	26;57	41;72	50;100	15	11	5
B4.1	9.65;37	24;57	39;72	50;100	19	14	2
B4.2	7.99;37	17;50	36;70	50;100	11	11	1
B4.3	14.02;37	24.4;57	35;70	50;100	3	3	0
B4.4	21.20;37	39;70	50;100		2	1	0
B5.1	22.29;37	32;60	41;70	50;100	3	4	1
B5.2	10;37	28;57	45.4;72	50;100	19	21	10
B5.3	14;37	25.9;57	36.4;72	50;100	10	12	6
B5.4	13;37	29;57	41;72	50;100	10	10	6
B5.5	16;37	31;57	42;70	50;100	11	22	11
B5.6	18.38;37	30;57	43;70	50;100	10	11	7
B6.1	16;37	32.2;57	40;70	50;100	8	10	3
B6.2	13.84;37	28;57	42;72	50;100	4	5	1
B6.3	15;50	34;70	50;100		0	1	1
B7.1	10.74;37	21;57	40;70	50;100	5	14	3
B7.2	15.28;37	26;57	35;70	43;90	4	6	4
B7.3	13;40	22;60	34;70	50;100	1	6	1
B8.1	9;37	20;57	38.6;72	50;100	17	20	2
B8.2	15.45;37	26.9;57	37.4;72	50;100	8	8	1
B8.3	16.49;37	28.7;57	42;70	50;100	15	40	10
B8.4	10.05;37	27;57	38;70	50;100	6	9	3
B8.5	17;40	30.5;57	38;72	50;100	7	11	4
BSP	1700;100				1	4	4
SB1	20.73;37	36.1;57	53;70	66;100	11	17	5
SB1	34;100				11	17	5
SB2.1	11.03;37	25;57	40;70	50;100	10	10	1
SB2.2	14;37	26.1;57	40;70	50;100	8	19	8
SB3.1	14.76;37	25.7;57	40;70	50;100	20	41	10
SB3.2	29;60	42;72	50;100		1	1	2
SB4	19.13;37	30;57	41;70	50;100	8	18	11

B. Equality and Diversity issues and breakdown of the results by gender

Table 3: Breakdown of results by gender

Class	Number								
	2020			2019			2018		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	18	55	73	13	46	59	9	49	58
II.1	28	38	66	18	49	67	15	52	67
II.2	3	10	13	5	15	20	9	16	25
III	1	3	4	1	3	4	0	2	2
P	0	1	1	0	0	0	0	2	2
F	0	0	0	0	1	1	0	0	0
Total	50	107	157	37	114	151	33	119	152
Class	Percentage								
	2020			2019			2018		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	36	51.4	43.7	35.14	40.35	39.07	27.27	41.18	38.16
II.1	56	35.51	45.76	48.65	42.98	44.37	45.45	43.7	44.08
II.2	6	9.35	7.68	13.51	13.16	13.25	27.27	13.45	16.45
III	2	2.8	2.4	2.7	2.63	2.65	0	1.68	1.32
P	0	0.93	0.93	0	0	0	0	0	0
F	0	0	0	0	0.88	0.66	0	0	0
Total	100	100	100	100	100	100	100	100	100

Table 3 shows the performances of candidates broken down by gender.

C. Detailed numbers on candidates' performance in each part of the examination

The number of candidates taking each paper is shown in Table 4.

Table 4: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
B1.1	28	38.14	7.99	63.64	13.53
B1.2	43	32.7	9.98	66.74	15.91
B2.1	33	38.94	6.05	72.39	9.96
B2.2	24	37.83	10.7	71.88	20.12
B3.1	54	28.39	8.39	68.83	11.13
B3.2	9	38.44	8.66	73	16.81
B3.3	9	36.89	6.74	68.33	8.89
B3.4	43	42.84	10.47	79.33	23.42
B3.5	31	34.94	10.01	67.55	15.06
B4.1	34	34.53	7.69	69.03	10.89
B4.2	22	33.18	8.58	70.29	10.91
B4.3	6	39.83	6.77	80.5	11.83
B4.4	3	44.33	4.73	84.67	13.05
B5.1	10	40.6	5.93	73.5	12.71
B5.2	51	38.45	9.68	70.59	15.84
B5.3	28	33.21	7.95	68.39	13.53
B5.4	27	36.44	7.43	68.15	11.59
B5.5	45	37.93	7.64	67.38	12.4
B5.6	30	39.67	9.24	72	17.06
B6.1	20	37.55	5.02	67.1	10.78
B6.2	11	37.18	7.18	68.09	9.96
B6.3	1	15		50	
B7.1	23	34.96	8.63	70.48	12.32
B7.2	15	31.07	6.78	64.87	12.39
B7.3	9	26.89	10.12	62.11	15.21
B8.1	29	33.55	8.21	69.48	10.88
B8.2	11	34.36	4.46	68.18	7.76
B8.3	47	36.45	8.38	66.19	13.52
B8.4	16	29.25	10.2	60.81	14.33
B8.5	26	33.15	7.88	63.77	14.39
SB1	2	23	7.07	49	0
SB2.1	7	33.86	9.6	68.86	16.19
SB2.2	17	34	9.06	66.35	13.05
SB3.1	55	33.76	8.41	64.69	12.42
SB3.2	2	44	0	79	0
SB4	19	37.79	6.55	68.47	12.24
CS3a	1	-	-	-	-
CS4b	2	-	-	-	-
BO1.1	7	-	-	68.71	6.21
BO1.1X	7	-	-	64.43	11.31
BN1.1	16	-	-	65.75	4.80
BN1.2	15	-	-	65.53	3.09
BEE	8	-	-	79.5	6.02
BSP	8	1200	158.88	70.5	9.44
101	1	-	-	-	-

Individual question statistics for Mathematics candidates are shown below for those papers offered by no fewer than six candidates.

Paper B1.1: Logic

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.63	21.63	2.57	27	0
Q2	17.92	17.92	5.62	13	0
Q3	15.69	15.69	6.10	16	0

Paper B1.2: Set Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.94	14.75	4.41	32	2
Q2	19.18	19.18	5.19	34	0
Q3	14.5	14.84	5.92	19	1

Paper B2.1: Introduction to Representation Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.25	17.25	4.19	28	0
Q2	20.79	20.79	1.81	14	0
Q3	21.29	21.29	3.01	24	0

Paper B2.2: Commutative Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.28	20.28	6.17	18	0
Q2	16.44	16.44	7.49	9	0
Q3	19	18.81	5.86	21	1

Paper B3.1: Galois Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.92	14.92	5.71	49	0
Q2	9.5	10	4.57	12	2
Q3	14.83	14.83	3.82	46	0

Paper B3.2: Geometry of Surfaces

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.78	19.78	4.74	9	0
Q2	19.33	19.33	5.51	3	0
Q3	18.33	18.33	5.05	6	0

Paper B3.3: Algebraic Curves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.5	18.5	3.12	8	0
Q2	14.4	14.4	4.34	5	0
Q3	20	22.4	6.10	5	1

Paper B3.4: Algebraic Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.69	13	8.09	10	3
Q2	21.37	21.37	5.17	35	0
Q3	23.51	23.51	3.94	41	0

Paper B3.5: Topology and Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.59	18.59	6.32	29	0
Q2	16.88	16.88	3.93	26	0
Q3	15	15	7.83	7	0

Paper B4.1: Functional Analysis I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.26	17.26	4.43	23	0
Q2	17.39	17.39	4.51	31	0
Q3	16.2	17	5.53	14	1

Paper B4.2: Functional Analysis II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.09	16.09	5.41	11	0
Q2	16	17	5.62	13	1
Q3	16.6	16.6	4.78	20	0

Paper B4.3: Distribution Theory and Fourier Analysis: An Introduction

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20	20	4.15	6	0
Q3	19.83	19.83	4.22	6	0

Paper B5.1: Stochastic Modelling and Biological Processes

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.17	19.17	3.82	6	0
Q2	23.14	23.14	2.04	7	0
Q3	18	18.43	4.38	7	1

Paper B5.2: Applied PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.17	18.30	6.11	46	1
Q2	21.52	21.52	3.99	46	0
Q3	12.73	16.125	8.08	8	3

Paper B5.3: Viscous Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.86	13.86	5.13	14	0
Q2	16.48	17.05	5.09	20	1
Q3	17.95	17.95	3.77	22	0

Paper B5.4: Waves and Compressible Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.56	15.56	4.57	18	0
Q2	21.08	21.08	3.27	26	0
Q3	15.6	15.6	4.30	10	0

Paper B5.5: Further Mathematical Biology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.23	19.23	2.79	44	0
Q2	18.95	18.95	5.19	38	0
Q3	17.625	17.625	6.30	8	0

Paper B5.6: Nonlinear Systems

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.61	18.5	5.61	22	1
Q2	18.5	19	5.73	19	1
Q3	21.24	22.21	5.23	19	2

Paper B6.1: Numerical Solution of Differential Equations I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.625	18.625	2.87	16	0
Q2	18.05	18.05	2.48	19	1
Q3	22	22	1.87	5	0

Paper B6.2: Numerical Solution of Differential Equations II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20	20	1	3	0
Q2	20.3	20.3	3.77	10	0
Q3	16.22	16.22	4.68	9	0

Paper B7.1: Classical Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.32	16.76	5.59	21	1
Q2	19.86	19.86	4.61	14	0
Q3	15.82	15.82	5.33	11	0

Paper B7.2: Electromagnetism

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.31	14.31	2.98	13	0
Q2	17.64	17.64	4.67	11	0
Q3	14.33	14.33	2.88	6	0

Paper B7.3: Further Quantum Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.5	10.5	5.43	6	0
Q2	13.6	16.25	8.73	4	1
Q3	14.25	14.25	4.77	8	0

Paper B8.1: Martingales through Measure Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.46	18.46	4.72	13	0
Q2	16.43	16.43	4.96	23	0
Q3	16.14	16.14	3.37	22	0

Paper B8.2: Continuous Martingales and Stochastic Calculus

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.7	18.7	1.06	10	0
Q2	15.27	15.27	4.41	11	0
Q3	23	23		1	0

Paper B8.3: Mathematical Models of Financial Derivatives

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.17	20.17	4.36	47	0
Q2	16.07	16.56	5.38	41	2
Q3	12.86	14.33	5.98	6	1

Paper B8.4: Communication Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.36	10.62	4.33	13	1
Q2	13.17	13.17	7.03	6	0
Q3	19.31	19.31	4.42	13	0

Paper B8.5: Graph Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.24	17.24	3.68	25	0
Q2	17.14	17.14	5.57	22	0
Q3	10.8	10.8	4.55	5	0

Paper SB2.1: Foundations of Statistical Inference

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17	17	4.58	7	0
Q2	16.86	16.86	5.34	7	0

Paper SB2.2: Statistical Machine Learning

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.42	16.09	5.70	11	1
Q2	13.29	13.29	4.86	7	0
Q3	19.25	19.25	4.89	16	0

Paper SB3.1: Applied Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.87	17.87	3.85	38	0
Q2	17.26	17.26	4.39	43	0
Q3	15.03	15.03	5.45	29	0

Paper SB3.2: Statistical Lifetime-Models

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21	21	-	1	0
Q2	22	22	1.41	2	0
Q3	23	23	-	1	0

Paper SB4: Actuarial Science

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.76	19.76	3.35	17	0
Q2	21.67	21.67	2.66	6	0
Q3	16.8	16.8	3.23	15	0

Assessors' comments on sections and on individual questions

The comments which follow were submitted by the assessors, and have been reproduced with only minimal editing. The examiners have not included assessors' statements suggesting where possible borderlines might lie; they did take note of this guidance when determining the USM maps. Some statistical data which can be found in Section C above has also been removed.

B1.1: Logic

Question 1 All candidates (but one) chose this question which is entirely on the first part of the course (propositional calculus) and which is heavily based on bookwork. Scores were typically quite high. For part (a)(iii) most candidates preferred the longish detour via proving adequacy of $\mathcal{L}_0$ over the much easier direct route via induction on n. In part (b)(ii) several candidates (unnecessarily) employed the Deduction Theorem without proving it. In the proof of CT (= Completeness Theorem, part (c)), a good number of candidates took the fact that any consistent set of formulas is satisfiable for granted while the proof of that fact is an essential ingredient in the proof for CT and should certainly not be omitted.

Question 2 Answers to part (a) and (b) were largely correct. Not many candidates have realised that the proof of the Soundness Theorem for the deductive system $K'(\mathcal{L})$ in part (c) is an easy consequence of the soundness of $K(\mathcal{L})$. Most candidates came up with the correct guess that $K'(\mathcal{L})$ does not satisfy the Completeness Theorem, but only very few candidates managed to present a convincing argument for it (which is, indeed, a rather challenging task).

Question 3 Some candidates had difficulties with part (b)(ii) and only a third of the candidates who attempted part (c)(iii) found a correct way for applying the Compactness Theorem. In part (c)(iv) many candidates immitated the proof for the isomorphism of two countable dense linear orderings without realising that for two countable models of Σ the argument for surjectivity does not carry over. Several candidates did not spot that the second part of (c)(iv) called for an application of the Loewenheim-Skolem Theorem.

General Impression: On the whole, the quality of the answers was higher than in previous years, obviously partly due to the fact that this was an open book exam, but maybe also because students had more time for revision during the lockdown. Typically candidates displayed a very thorough understanding of the material.

B1.2: Set Theory

Question 1.

Part (a) subparts ii, iii, iv were generally well done with clear demonstrations. Also most found simple counter-examples to i. A few submitted fallacious proofs of i, or of iv. Part (b) is pure bookwork and most solutions were fully satisfactory. Part (c) subpart i was a challenging problem requiring good understanding and care to formalise the argument. Many simply quoted the main theorem on comparison of well-orders from the notes and did not make any real progress. Some saw what needed to be done but did not formalise the construction. Quite a few however gave full and correct solutions. Subpart ii is fairly easy from bookwork (assuming i) and was well done. Subpart iii required a good understanding but was well done by many.

Question 2.

Part (a) subpart i was generally well done. Also ii was handled successfully by many, though while quite concise proofs can be given based on standard results, many proofs were quite circuitous, with the circle sometimes not closing. Subpart iii was generally well done. In part (b), subparts i and iii were generally well done, while ii less so as several proofs failed to deal with all possibilities. Part (c) is very much along the lines of many past problems, and the construction of the transitive closure (in a problem set), and was generally well done with no issues. Quite a few forgot to show the uniqueness.

Question 3.

In part (a), subparts i and ii were generally well done. In subpart iii some of the proofs were less clear. Part (b) is pure bookwork and was generally well done. Part (c) was more difficult. The fact that, if the conclusion fails, one generates an infinite descending chain, was just the first insight, and a careful proof setting out how this gives a contradiction, using axiom of choice or otherwise, was needed. Some attempted proofs by induction were not correct. Quite a few however gave essentially complete answers.

0.1 B2.1 Introduction to Representation Theory

Question 1. (a) (i) Some candidates forgot to say that the identity is in A. (This is important later, as the problem uses properties of algebras with identity.)

(iii) The candidates were expected to give a short justification (corollary to Jordan-Holder) why all the simple modules can be found just by looking at the composition series they found in (ii).

(b) (ii) One mistake here was to say that if S is a simple module and $x \in B$, then $x \cdot S$ is either S or 0. This is false since $x \cdot S$ is not a submodule in general (but it shouldn't be needed for the argument).

(iii) We emphasise that $A/\mathtt{rad}(A)$ is isomorphic to k^2 as an algebra (this is the reason why one checks that the radical is a two-sided ideal).

(c) This was the most difficult part of the question. The answer is that A is isomorphic to its opposite via the map that swaps a and b on the diagonal.

Question 2. The solutions were generally correct. One common mistake was in (b) (i), where some candidates did not argue that $g^{-1}a$ is in U, which is a finite group, and therefore $\overline{\widetilde{\chi}(g^{-1}a)} = \widetilde{\chi}(a^{-1}g)$. This is an important point, that's why the assumptions about U are needed.

Also for (b)(iii), one needs to consider the subgroup U generated by G an a, argue that it's finite and then apply (ii).

Question 3. The solutions were generally correct. In (a)(ii), one needs to say that characters determine the representations uniquely (since we are over $\mathbb{C}$), as the typical proof went by checking that the relevant characters are equal. Some common mistakes were found in (a)(iii): the simplest solution would use Frobenius reciprocity and an irreducible subrepresentation of the restriction of V to H. It is also possible to use the group algebra $\mathbb{C}G$ and induction from the group algebra of H. For the character table of A_4, when determining the one-dimensional characters via their values on (123) and (132), one needs to argue why these give well-defined representations (a priori it is possible that not all values are consistent, as it happens for example for the dihedral groups). The best way is to use the abelianisation of the group, which is C_3 and lift characters. Also, if one uses the standard representation of S_4 restricted to A_4, one needs to argue that the restriction is irreducible.

B2.2: Commutative Algebra

Question 1: This was a popular question, chosen by a majority of the students. Most people managed to solve the unseen part (d) correctly, although some were worried that further hypotheses on q than those given were necessary, this is in fact not the case. There is, in fact, a nice alternative solution to part (d) using just linear algebra in place of the Nullstellensatz.

Question 2: This was the least popular question, with few students managing to find a correct solution to the unseen part (d) of the question. *Normalisation* of affine varieties (a form of resolution of singularieties) is not on the syllabus of B2.2, but Q2(d) gives an excellent example of a situation where the normalisation of a connected affine variety fails to be connected: here, two affine lines meeting at a point are pulled apart by the normalisation process. This geometric picture is made possible by the existence of the new non-trivial idempotents $x/(x+y)$ and $y/(x+y)$ in the ring of fractions.

Question 3: This question was universally popular, and was mostly done well. Most marks were lost by students not having a full grasp of the logic required to do part (d). The Going Up Theorem, part (c), admits two different-looking proofs, one using localisation and the other one - not. Most students gave the second solution, but it was easier to lose marks this way because several necessary details could easily be overlooked.

B3.1: Galois Theory

The distribution of raw marks was very homogenous for this exam.

Question 1.

This question was quite popular. The answers were generally satisfactory but many students forgot to prove some of their intermediate results, especially the irreducibility of certain inseparable polynomials.

Question 2.

This question was the least popular. In (d), many students forgot to prove that in order to prove that κ is well-defined, it is necessary to note that G_M is a normal subgroup of $\mathrm{Aut}_{\mathbf{Q}}(L)$. This follows from the fact that M is a Galois extension of $\mathbf{Q}$ and the fundamental theorem of Galois theory. Very few students solved (d), (e) and (f) completely. In (c), many students made a mistake because they did not carefully check that the decomposition $g = k(g) \cdot m(g)$ they proposed really worked.

Question 3. Along with question 1, this was the most popular question. Many students did not think of applying Artin's lemma or its variants (Theorem in section 5.2 of the notes) in (a) (ii). Most students answered (b) satisfactorily. In (c), many students attempted to apply an extension theorem for embeddings of fields, which cannot be applied here because its assumptions are not satisfied. In their solutions to (d) (i), many students asserted that $[K(\mu_p) : K] = p - 1$, forgetting that K is not necessarily $\mathbf{Q}$! In (iii), a lengthy elementary solution was often provided, instead of a shorter one, which relies on (i) and (ii).

B3.2: Geometry of Surfaces

With the exception of the very weakest candidates, students tended to get full marks, or very nearly full marks, on the bookwork parts of the questions 1(a),2(a),3(a), and elsewhere. I also noticed several occasions on which candidates had completely scrambled working but then miraculously ended up with the correct (unknown) answer at the last minute, e.g. that for 3(b)(i) the answer should be a catenoid. I take it these are the effects of open book exams, and being able to do a google search.

In view of the open book format, the questions and mark scheme had been adjusted to allocate fewer marks to bookwork than usual, and more to parts which required understanding of the material. This may have resulted in a greater spread of marks than in a normal year, with weaker candidates unable to pick up as many marks as usual for bookwork and memorization.

The parts candidates found most difficult were:

1(d) In principle this is an elementary counting problem building on the preceding bookwork, but candidates tended to write things like the number of ways of writing $c = a + b$ with $a, b \geq 0$ is c, not $c + 1$. Also you have to understand that given $X\#Y$ is orientable/non-orientable, what are the possibilities for X, Y orientable/non-orientable?

3(b)(iv) This is about understanding what the graph of a function $f(u)$ satisfying $f(u)^2 + (1 + f'(u))^{-1} = c$ looks like. For $c > 1$, the correct graph is U-shaped over $[a, b]$, with $f = c$ and $f' = \infty$ at $u = a, b$, and $f = \sqrt{c-1}$ and $f' = 0$ at $u = (a + b)/2$. You can convince yourself that there is a turning point when $f' = 0$, rather than an asymptote as $u \to \pm\infty$ (as happens when $c = 1$), by showing that $f'' > 0$ when $f' = 0$. Only one candidate got the right picture.

B3.3 Algebraic Curves

Question 1: Candidates largely did well in the computational part although there was a substantial number of inaccuracies such as "division by zero". In c), many candidates used arguments involving excess intersection without carefully showing that those intersection points are really distinct. In e), although most candidates were able to write down an explicit bijection, in the proofs there were some excessive appeals to geometric intuition leading to a lack of rigour and sometimes even to incomplete arguments.

Question 2: In a), some candidates forgot that an inflection point also has to be nonsingular. In b), almost all candidates used an incorrect argument involving Bézout's theorem to show that there are only finitely many inflection points on a curve, neglecting the possibility that it is a component of the Hessian. In f), most candidates had the correct approach but computational errors led to a few wrong results. Rather noteworthy, the ramification indices computed by some candidates were off from the correct result by exactly one.

Question 3: Almost all candidates produced their best results and the level of answers was very high. In f), some candidates incorrectly assumed it was enough to show that $\ell(nP) > 0$ ($\ell(nP) > 1$ was necessary) while others did not prove that the function obtained really has a pole at nP: being in $\mathcal{L}(nP)$ only gives an upper bound on the pole order at P.

B3.4: Algebraic Number Theory

The exam seems to have been found generally quite straightforward. Quite a lot of the material - particularly the first half of Q2 and all of Q3 - were of a standard type.

Q1 saw relatively few attempts. It was of a fairly standard type, related questions having appeared on previous exams. The final part (f) was somewhat novel and this seems to have caused the most difficulty to candidates.

Part (a) of Q2 was very standard and many candidates solved it easily. Part (b) was novel. It was quite well-answered, with a number of different solutions. One which caught my eye particularly was the observation that (2020) has six distinct prime factors in $\mathbb{Q}(\sqrt{-79})$, precisely the same field considered in Q3!

Unfortunately the examiner made an error in setting Q3, taking the Minkoswki bound to be double what it actually is. The effect of this is that the question is easier than intended. Thus in part (a), where candidates were asked to show that $M_K < 13$, we in fact have $M_K < 6$. Consequently in part (d) one only need examine the primes 2,3,5 and not 7 and 11. This renders part of the hint (the suggestion that candidates consider the factorisation of $3 + \sqrt{-79}$) irrelevant. This had the potential to confuse candidates but I did not see very much evidence of this. A few did consider the factorisations of (7) and (11) as well "just in case" and I apologize to them.

B3.5 Topology and Groups

There were 42 attempts. Of these only 7 achieved less than half of the marks available. The remaining candidates showed generally a good understanding of the subject. Most candidates did not attempt Question 3 concerning covering spaces.

Question 1: (40 attempts) Part (a) and (b) were well done. Part (c) proved more challenging. Points were lost when it was not made explicit that the function is well-defined, a most important point here. Also, candidates let themselves get confused by trying to follow a proof for the Fundamental Theorem of Algebra in the lecture notes which is somewhat different. All in all the question was well done with an average marks around 20.

Question 2: (37 attempts) This was quite a demanding question, drawing on different parts and techniques of the course. Many were able to compute the fundamental group of (a)(i) realising that the resulting space is a wedge product of the original space with a circle. The second space in (a) (ii) proved much harder. One way to solve it would have been to see that the resulting space is homotopy equivalent to the original space wedged with a copy of the circle and the sphere but no candidate did so. Some good attempts were made using the Seifert-van Kampen Theorem directly but often were flawed, for example when the intersection of the two subspaces was not path-connected. Part (b) of the question saw some good attempts with most students able to write down the required maps and showing well-definedness of the maps in and out of the fundamental group of the Klein bottle. Hardly any students proved injectivity of the map from the integers by noting that composition with the second map produces and isomorphisms. Most challenging was the last part, to find the appropriate continuous maps. But that too saw some very competent solutions. All in all, the question produced a good spread of marks.

Question 3: (6 attempts) It was a bit disappointing to see so few candidates approaching this question as it covered a significant and important part of the course. There were no particular surprises. The questions themselves being mainly straight forward and produced a good spread of marks, including some very high ones.

B4.1: Functional Analysis I

Question 1 has been solved by about two thirds of the candidates. Part a) was very well solved with many candidates getting full marks. In part b) most candidates realised that they could adjust examples used to show the incompleteness of the space of continuous functions equipped with the L^1 norm to establish incompleteness of Y, but quite a few struggled with the first part of b)i) as they worked with inapproptiate notions of convergence (such as pointwise) rather than exploiting that if (f_n) is Cauchy sequence in X then $\frac{f_n}{x}$ is a Cauchy sequence in L^1 or arguing that the given space is isometrically isomorphic to L^1 via $f \mapsto \frac{1}{x}f$. Part (ii) was designed to be somewhat challenging and as expected not all students realised that they had to combine the norms of the two spaces X and $C(I)$ somehow (e.g. taking the sum of the norms) to obtain a suitable norm on Y, so that any Cauchy sequence in Y will be Cauchy in both X and in $C(I)$ and even fewer students realised that since they are considering two different notions of convergence they need to comment why the obtained limits agree. In part

c) most students scored at least some partial, though not all of the given examples had both the required properties (or in some cases were even a subspace).

Question 2 was solved by most of the candidates. The first three parts of a) were variations or applications of bookwork. Part (ii) caused more difficulty than expected with some students confusing pointwise convergence of operators with convergence in norm, some candidates using the assumption of Cauchy sequence already in (ii) where this is not assumed and some solutions making improper use of $\lim inf$. Part (iv) was designed to be more difficult, but many students got the right idea of using a non-convergent Cauchy sequence in the space Z to build a non-convergent Cauchy-sequence in the space $L(X, Z)$ using e.g. the element of $X^* \setminus \{0\}$ from (i). Part b) was a standard application. The second part was quite easy and many students got full marks, while the first part required more care.

Question 3 was solved by slightly less than half of the students. The first part was part of a exercise from a problem sheet and most students got high marks on that, though some lost points as they were not careful enough in arguments that involved infinite sums or limits. Part b), which was an application of bookwork, was generally well solved. Most students correctly explained that while quite a few of the assumptions of Stone-Weierstrass are violated (including the assumption that L needs to be a subspace that many students forgot) the set is still dense and gave good proofs arguing either that the closure of the set needs to contain all piecewise linear functions, which in turn are dense by Stone-Weierstrass, or explaining carefully why the proof of Stone-Weierstrass still works with the given set L. Part c) was a standard application and generally well solved, though a few students tried to use arguments from the seen case where this operator is considered on the space of continuous function, such as trying to disprove surjectivity by saying that each element of the image has value 0 at 0, that do not apply in the setting of L^p functions. The last part was designed to be a bit more difficult than other parts of the question but many students came up with good approaches, such as considering a dense not closed subspace Y of a Banach space X in a).

B4.2: Functional Analysis II

Question 1. This question was tried by about a half of the candidates. Parts a)-c)i) were handled reasonably well with minor exceptions. Only a few candidates were able to tackle c)ii) and d) successfully. There are different ways to handle these. One easy solution for c)ii) is to realise that, if $A_n \to I$ in norm, then A_n is invertible for large n, which is impossible as the range of A_n is a proper subspace. For d), one may first show, for example, that A_n is the orthogonal projection onto V_n and then use the density of the set of polynomials in X.

Question 2. This question was tried by about two thirds of the candidates. The standard parts of the question were handled well with minor exceptions. Most candidates had some ideas on how to handle c) but lost a mark here and there somehow. A number of candidates used a sophisticated argument involving the principle of uniform boundedness to show the boundedness of K, as opposed to an easy application of Cauchy-Schwarz' inequality. Only a handful of candidates were successful in using d)i) and a 3ε-argument to finish d)ii).

Question 3. This question was attempted by all but three candidates. Overall the question was handled reasonably well with minor exceptions. Most candidates finished only half of c)ii), and only half of the candidates had a feeling on how to handle c)iii). Many candidates forgot that in c)ii), one only needed to consider real λ's. The case $|\lambda| \geq 2$ is more or less straightforward. When $|\lambda| < 2$, k_1, k_2 are complex, unimodular and conjugate to one another. Hence the perspective eigenvector is of the form $(\alpha \sin n\theta)_{n\geq 1}$, for some constant α and θ, which is readily seen not to belong to ℓ^2 unless $\alpha = 0$. A truncation of this sequence could be used to construct approximate eigenvectors in c)iii).

B4.3: Distribution Theory and Fourier Analysis: An Introduction

There were only 6 candidates taking the exam and their performances were generally quite good. The marking scheme was used throughout.

Question 1: All did well and got full marks for (a) and (b). The marks for these had been reduced for the occassion as large parts are bookwork and material that can easily be found in the lecture notes. In fact, there is an unfortunate typo in (b) (a minus sign is missing). However it didn't cause any disruption as it doesn't affect later parts of the question and only two candidates mentioned it (and commented that the minus sign was correctly there in the lecture notes). In part (c) half of the candidates lost marks, but it isn't clear what they found difficult here as some lost marks on the first half and some on the second half of the question. One candidate explicitly mentioned 'time constraints' and then passed onto part (d) of the question. For part (d) only two candidates obtained the full 7 marks. It is an application of the result found in (c) and the last part is a straightforward application of distributional derivatives in the context of compactly supported distributions.

Question 2: Nobody attempted this question.

Question 3: All did well and got full marks on (a). The marks for this bookwork part had been reduced for the occassion. Part (b) also went well for most candidates and consisted of a variant of a problem from a problem sheet. Nobody got full marks for part (c), though 3 candidates got very close. The ones who didn't do well on this part hadn't realised that the distribution was the second derivative of the principal branch of the complex logarithm. Examples of a similar kind had been on problem sheets. Part (d) went better than I had anticipated, and those who didn't get full marks had difficulties finding a particular solution.

Recommendation: Out of the 50 raw marks,

- First class performance: > 35
- Upper second class performance: $[28, 35]$

B4.4: Fourier Analysis and PDE's

There were only a few candidates taking the exam and they all performed well. The marking scheme was used throughout. I suggest that no scaling is applied.

B5.1: Stochastic Modelling and Biological Processes

All candidates achieved good exam results, with the average raw mark being 81%. This was a significant improvement comparing with the 2019 exam paper, when the average raw mark in the B5.1 exam was just short of 40%. For the benefit of students considering to take the course B5.1 in Hilary 2021, it is worth mentioning that the jump in performance from 40% to 81% was not caused by a change in the course syllabus. The course B5.1 or its examination did not become in any way easier.

In 2019, the examiner's report mentioned that "there was a significant number of incomplete and incorrect solutions, which worryingly showed gaps in some candidates' understanding of background Prelims and Part A courses, which the course B5.1 builds on". We are glad to report that none of the candidates who took the B5.1 exam in 2020 would fall into such a category.

In 2020, only 1/3 of the candidates took the B5.1 examination, so one would get a better comparison of the 2019 and 2020 years, when comparing the 2020 results with the top 1/3 of the course cohort in 2019. Then the result gap would be diminished. In Hilary 2021, it will again be important that students who register for the course B5.1 have good understanding of the background Prelims and Part A courses that the course B5.1 builds on.

All three questions in the 2020 exam paper were attempted by a similar number of candidates. To solve Question 1, candidates used the Fokker-Planck equation together with techniques for calculating the average time and probability of adsorption. Candidates showed a very good understanding of the material with the average raw mark for Question 1 being 19.2 (to put it into context, the exam question testing the same part of the syllabus in the 2019 exam only averaged at 7.4 raw marks).

In Question 2, the candidates used the Laplace transform with confidence (covered in the prerequisite Part A course Integral Transforms). In Question 3, it has been a pleasure to see that candidates applied a number of different methods for studying stochastic chemical reaction systems and found alternative ways to solve parts (c) and (d). Some candidates derived and solved a PDE for the probability generating function, some used the chemical master equation to obtain the inequality for $\psi(t)$ and some derived a closed formula for $\psi(t)$.

B5.2: Applied PDEs

Question 1

Q1a) was generally well done, except for candidates occasionally committing omissions and sloppinesses.

Q1b) The Burgers' equation is a standard example for quasilinear PDEs. Unsurprisingly, students did well with determining the criterion for the schock speed and causality; only a few dropped marks for giving prose when answering the latter rather than an explicit condition in terms of u_- and u_+.

The initial data was quite complicated, giving rise to three phases: Phase (1) involved only continuous solutions and lasted up to $t = 1$. Most students got this very well, but some failed to write make the domains in (x, t) where each case of the solution was sufficiently clear. At $t = 1$ a shock forms, with flat adjacent states, hence constant velocity. This, too, was correctly obtained by the vast majority of students, with similar limitations regarding the presentation of the solution. These two phases were covered in (b)(ii) and many students did well here.

The next phase, after $t = 3$, involves the transition to a solution with a curved shock trajectory, and this is where students had difficulties. However, a large group of student did well here, too (question (b)(iii)).

Question 2

2(a) - bookwork - was generally well done, except for candidates occasionally committing omissions and sloppinesses.

2(b)(i) Also worked well for most students, with a large majority getting the correct answer for α.

2(b)(ii) A large number of students also managed to obtain a solution for u. The level of required simplification was not specified in the question, so the main error that could be made here was algebraic, which quite a few students committed. Another error that was occasionally made was to remember that the formula for u involves line integrals.

Question 3 This question was rarely done. Some of those who did struggled at various stages:

Q3(a)(i) was not really bookwork but a variation on the Dirichlet problem that was extensively discussed in the course. Some students did not know how to handle this.

Q3(a)(ii) The major hurdle here, it appeared, was to realise that the conserved quantity I fixes one of the two similarity exponents. Some students just guessed a condition or inappropriately tried to obtain the information from the boundary condition. This part was one of the difficult parts of the question.

Q3(b)(i) required understanding that the superposition of K guarantees that the PDE is fulfilled and also the initial condition, but only one of the two boundary conditions. Students who realised this finished this part quickly.

Q3(b)(ii) With few exceptions, candidates were somewhat baffled by the question, muddled the algebra required to showing (6) or did not know how to obtain the limit.

This question catered to candidates who spotted the idea of the different parts quickly, as the required computational effort was moderate.

B5.3: Viscous Flow

Most candidates showed a good understanding of the material. Every question attracted a few near-perfect solutions, though many candidates found the last parts of questions challenging. Question 1 was least popular, attempted by roughly half the candidates. Questions 2 and 3 were equally popular, and attempted by 80% of candidates.

Question 1

Part (a) was done well by nearly all candidates, though a few did not notice that $\mathbf{F}$ is the force per unit volume, not the force per unit mass.

In part (b) the drag force is in the along-stream direction, equal to the integral of $-\mathbf{e}_x \cdot \boldsymbol{\sigma} \cdot \mathbf{n}$ around the boundary, with $-\mathbf{n}$ pointing into the fluid. Evaluating $\boldsymbol{\sigma}$ for $\mathbf{u} = u(y,z)\mathbf{e}_x$ and $\mathbf{n} = (0, n_y, n_z)$ gives the required result. Some candidates did not justify the disappearance of σ_{xx} and the pressure in this geometry.

In part (c) candidates were expected to identify the velocity scale $U = FL^2/\mu$ that reduces the x-component of the Navier–Stokes equation to $\hat{\nabla}^2\hat{u} = -1$, so $-\int_{\partial\hat{D}} \mathbf{n} \cdot \nabla\hat{u}\,\mathrm{d}s = -\iint_{\hat{D}} \nabla^2\hat{u}\,\mathrm{d}y\mathrm{d}z = \mathrm{area}(\hat{D})$. Some candidates tried to scale the drag force instead.

In part (d) only three candidates made substantial progress. A few candidates were caught out by minus signs when integrating with respect to arclength around the boundary. The contributions to the drag from opposite edges are equal. Combining the contributions from all four edges and simplifying gives a sum that factorises into a sum over n and a sum over m, each of which can be evaluated using the hint.

Question 2 was most popular.

Part (a) was mostly done very well, though a few candidates did not use the behaviour of the flow as $Y \to \infty$ to to justify discarding the $\partial_x p$ term in the equation for the streamfunction.

Part (b) most candidates used incompressibility and integration by parts to show that $\mathrm{d}M/\mathrm{d}x = -\mathrm{d}M/\mathrm{d}x$, and so must vanish. Several candidates lost factors of 2 in reassembling their integration by parts into $-\,\mathrm{d}M/\mathrm{d}x$.

Part (c) almost all candidates found $\alpha + \beta = 1$ to balance the powers of x and leave an ODE for $f(\eta)$. However, only four candidates used constancy of M from part (b) to find the second condition $2\alpha = \beta$ that leads to the unique solution $\alpha = 1/3$ and $\beta = 2/3$. Many candidates reverse-engineered these values of α and β to arrive at the ODE given in the question.

Part (d) most candidates showed that the given expression solved the ODE, some by integrating the ODE twice using the boundary conditions, others by differentiating the given expression. Most candidates calculated $M = (9/2)f_\infty^3$, though several lost a factor of 2 when evaluating the integral over $(-\infty, \infty)$ using the hint for an integral over $[0, \infty)$.

Few candidates even attempted the sketch. This should show an expanding jet with streamlines of the form $Y \sim x^{1/3}$ for large x (there is no finite asymptotes for the streamlines). The jet is fed by a small vertical inflow that is strongest near the Y axis. The horizontal velocity and pressure perturbation vanish as $Y \to \pm\infty$, but the inward vertical velocity tends to a small Y-independent function of x. A few candidates drew good sketches.

Question 3 was the next most popular

Parts (a) to (c) were mostly done well, though several candidates used incorrect kinematic conditions on the free surface in part (b), and a few did not correctly adapt the treatment in lectures to 3D.

Part (d) caused more difficulty, mainly with converting the evolution equation for $\hat{t}$ from part (c) into polar coordinates. This is easiest using the formulae in the hint to express $\nabla \cdot (\frac{1}{3}\hat{h}^3 \nabla \hat{h})$ in polar coordinates. The resulting equation for $\hat{h}(\hat{r})$ can be integrated twice with respect to $\hat{r}$ to find the steady solution. The maximum velocity occurs at the free surface, as found in part (c), giving $\hat{u}_{\hat{r}} = -\partial_{\hat{r}}(\hat{h}^3/6) \sim 1/(8r_0)(6Q/\pi)^{3/4}\delta^{-1/4}$ as $\delta \to 0$.

B5.4: Waves and Compressible Flow

Question 1: The bookwork in part (a) was very well done. A minority of candidates lost marks by claiming incorrectly that $(\rho_0(z)w'_{zt})_{zt} = \rho_0(z)w_{zztt}$ and then going on to derive erroneously the correct equation for w'. The derivation of the solutions for the wave-maker in part (b) were also very well done. A small minority of candidates lost a mark for a sign mistake in the solution of the ODE for the x-dependence of w'.

The derivation of the linearized boundary condition for w' in part (c)(i) was well done by about half of the candidates. The remainder lost marks for incorrectly applying Taylor's Theorem to the dynamic boundary condition or for incorrectly eliminating p' and η. Only a handful of candidates derived the correct dispersion relation in part (c)(ii) by solving for λ, and hence ω, each as a function of k, so this tail was on the harder side.

Question 2: This was the most popular question, attempted by virtually all candidates. The fairly routine exercise in separation of variables in part (a) was generally handled well, although many failed to notice that the Neumann boundary conditions in the x-direction permit the constant solution, in contrast with the Dirichlet boundary conditions in the y-direction. The very few students who seemed to be put off by the slightly misleading wording ("pressure perturbation" instead of "perturbed pressure") were given credit for otherwise correct working.

Part (b) involved a relatively straightforward Fourier transform and stationary phase calculation which again was generally done well. The main difficulty encountered was with differentiating functions like $|k|^3$, and quite a few candidates also didn't realise the significance of ϵ being small.

Question 3: This was the least popular question. The standard dam-break problem in part (a)(i) caused few difficulties apart from an occasional lack of sufficient explanation. On the other hand, very few candidates made any serious headway with part (a)(ii): a frequent error was to try and solve for positive characteristics $\mathrm{d}x/\mathrm{d}t = u + c$ instead of the particle paths $\mathrm{d}x/\mathrm{d}t = u$.

Part (b)(i) just required straightforward algebraic manipulation of the given algebraic Rankine–Hugoniot equations, which nevertheless many students struggled to perform accurately. In part (b)(ii), most students were able to derive the general formula for energy production, but again were then defeated by the simple algebraic steps needed to obtain $q(\beta)$ correctly.

B5.5: Further Mathematical Biology

Question 1: This question was a popular choice, with all candidates but one choosing to answer it. Overall, candidates answered this question well. In part a, many candidates were confused by the rate of diffusion of u being proportional to the density of v. Bookwork in parts b and c was completed successfully (as expected for an open book exam!) In part d, the application of the theory from parts b and c generally started well, but only a few candidates completed the final part of the question.

Question 2: Most candidates answered this question. The nondimensionalisation in part a posed few problems (other than some small algebraic slips). The reduction based on asymptotics in part b was carried out very well by most candidates, although some candidates failed to get started entirely. Part c was essentially bookwork, and so candidates answered this part of the question successfully. Fewer candidates answered part d correctly, with some attempting a phase plane analysis rather than solving the DE provided directly (which was disappointing, given the clear instruction in the question) – however, lots of candidates provided a comprehensive answer to this part of the question and scored full marks on it (perhaps reflecting that a similar question was on a problem sheet).

Question 3: Only a few candidates chose to answer this question (with ten submissions in total). Around half of those candidates submitted answers that were largely correct (aside from minor slips). Parts a (model interpretation) and b (non-dimensionalisation and solution of approximate quasi-steady equation) were completed with few problems. Parts c-e (analysis of the model, and interpretation of the results of the analysis) were generally where most marks were lost.

B5.6: Nonlinear Systems

There were a lot of good answers to each of the questions. Even answers which didn't score well usually lost marks through algebraic mistakes or sloppiness rather than through a lack of understanding.

In Q1. a common mistake was to change to canonical variables $\tilde{x}$, $\tilde{y}$ in order to find the centre manifold, to drop the tildes, and then to use the original equation for $\dot{x}$ to determine the dynamics rather than the equation for $\dot{\tilde{x}}$. Another sloppy mistake repeated on more than one script was to calculate the equations for $\tilde{x}$ and $\tilde{y}$ carefully in part (a), and then [again because of dropping the tildes] simply to add $\mu\tilde{y}$ to the right-hand side of $\dot{\tilde{x}}$, rather than recalculating the transformation of the new equations [μ will appear on the right-hand side of both $\dot{\tilde{x}}$ and $\dot{\tilde{y}}$].

In Q2. most errors were algebraic mistakes, though there was some confusion as to what constituted a period-2 bifurcation from an unstable steady state.

In Q3. there were a lot of good answers easily identifying the homoclinic orbit explicitly, spotting that $u = 1$ muts be a double root of the cubic expression which arises. A common mistake was to use $\leq$ rather than $<$ in the range of δ values in the final part of the question, missing the fact that at the end of the range of δ the zero will be a double zero rather than the required simple zero of $M(t_0)$.

B6.1: Numerical Solution of Differential Equations I

Q1. The question concerned the convergence analysis of the implicit Euler method, as an example of a one-stage implicit Runge–Kutta method. In part (a) of the question some of the candidates erroneously stated that the required Lipschitz condition, with Lipschitz constant equal to 1, was the consequence of $|y''(x)|$ being bounded by $2/(3\sqrt{3}) < 1$. In part (b) of the question some of the candidates failed to justify that the mapping $y \in \mathbb{R} \mapsto y - hf(y) \in \mathbb{R}$ was surjective (onto). Part (c) of the question was generally well done, although some of those who attempted this part failed to provide a convincing justification of the existence of an $h_0 \in (0, 1)$ such that the bound $|y(x_n) - y_n| \leq 10^{-2}$ holds for all $x_n \in [0, 1]$ and all $h \in (0, h_0]$.

Q2. This question was attempted by all candidates who sat the exam. Parts (a)–(d) were generally well done, but many of the candidates had difficulties with part (e) of the question, and embarked on tedious and long-winded calculations based on Schur's criterion (or the Routh–Hurwitz criterion) to show that the stability polynomial $\pi(z, \bar{h}) := (1 - \bar{h}b)z^2 - \bar{h}z - (1 + \bar{h}(1 - b))$ of the linear multistep method was not a Schur polynomial, failing to observe that $z = -1$ was one of the roots of this polynomial which then directly implies the required absolute instability for all $b \in \mathbb{R} \setminus \{1/\bar{h}\}$. Only one candidate correctly realised that the case of $b = 1/\bar{h}$, when the quadratic stability polynomial collapses to the linear polynomial $\pi(z, \bar{h}) = -\bar{h}z - \bar{h}$, with unique root $z = -1$, needed to be considered separately.

Q3. This question, concerning the proof of a (conditional) maximum norm error bound for the θ-scheme approximation of a parabolic initial boundary-value problem, was generally well done by the six candidates who attempted it, although several of them lost marks by failing to state the initial and/or boundary conditions for the scheme, or failed to specify (or incorrectly specified) the range of indices in the spatial and/or temporal direction for which the scheme was being considered.

B6.2: Numerical Solution of Differential Equations II

This exam had a rather bimodal distribution of marks, with many high scores. It is unclear exactly the effect of it being sat as an open book exam, but certain parts must have been easier because of this.

The first question on 2-point boundary value problems was attempted by only a handful of candidates who mostly produced good solutions, except for the final part where the Jacobian of a nonlinear system of finite difference equations was asked for. Only one candidate was able to derive this successfully.

The second question on finite difference approximatin of a variant of the Poisson model problem was attempted by all but one candidate and there were many high scores. Some candidates did rather more than was required, but most who presented a carefully argued solution did well. Some observed that the continuous Neumann problem for the Laplacian has a kernel consisting of constant functions, but failed to identify that the discrete problem also has any vector of constant values in the corresponding matrix kernel.

The third question on the Lax-Freidrichs and Lax-Wendroff methods for first order wave equations was generally reasonably done, though there was more carelessness in writing down some arguments/calculations. There was some confusion about stability and, in particular, the necessity of the CFL condition in this regard. Only a small number of candidates correctly observed that the higher order accuracy of the Lax-Freidrichs scheme for the modified equation in the final part might imply dissipativity.

B6.3: Integer Programming

Only a small number of students took the paper. Despite the open book format, the spread of marks was similar to previous years, which suggests that the paper worked well in this format. All three questions were attempted, and the marks achieved by each student on the two questions they selected was within 3 marks in each case, which suggests that the problems were roughly of equal difficulty.

Question 1 tested students' understanding of LP relaxations, Chvàtal-Gomoroy cuts and Fourier-Motzkin elimination, which makes a connection between the early and late parts of the course. This problem was solved very well.

Question 2 tested LP based branch and bound in the context of the binary knapsack problem, the notion of a relaxation of an IP, and the notion of group relaxation for equality constrained integer knapsack problems. There was a good range of marks, the top being close to full marks.

Question 3 explored the notion of total unimodularity, and the technique of Lagrangian relaxation in the context of a particular IP for which each Lagrangian subproblem is totally unimodular and hence easy to solve. The range of marks was a bit larger on this question, despite it being a fairly standard problem.

Overall, the paper worked well and seems to have given the students the chance to prove themselves.

B7.1: Classical Mechanics

The questions in general seemed to work well in spite of the new open book format.

Questions:

1. Nearly all candidates attempted this question. Parts a and b were by and large well done, although there was some confusion as to how to use the conserved quantities from the boosts to solve for the centre of mass (its simplest to also know the conservation of total momentum). Many candidates had forgotten the correct procedure to obtain an effective Lagrangian, i.e., to correctly match equations of motion.

2. This was a routine question on normal modes that also required some expertise in rigid bodies. This caused some difficulty for some who ignored the angular motion of the rod or didnt see how to incorporate that of the square lamina. The last part was relatively routine and well done with few errors.

3. The basic material on Poisson brackets and the Legendre transform was mostly well done. Although many noted that the angular mo-

menta are conserved quantities for the motion, no one spotted that their conservation already directly gives the orbits as great circles.

B7.2: Electromagnetism

Generally students did well. Most students seem to understand the main ideas and basic content of the lecture course. There where however many computational errors, in particular in the newer parts of the questions.

Question 1

This question is about electrostatics. There were 13 attempts and a number of them where excellent.

Parts (a) and (b) where well done. A few students did not use the fact that the electrostatic field is a gradient complicating the computations, and many missed the charge at the origin hence obtaining the incorrect total charge.

Part (c) Most students who attempted this part had the correct idea. There where many numerical errors when computing the potential . Only a couple of students could interpret correctly the terms in the multipole expansion.

Question 2 There were 11 attempts some of the excellent. This is a question about a magnetostatic configuration which can be solved using the tools learned to solve electrostatics problems. Most points where lost in parts (c) and (d) where some students stated incorrectly the boundary conditions or implemented them incorrectly.

Question 3 This question is about electromagnetic wave traveling between two plates. There were 6 attempts. Students had many problems in this question stating the correct boundary conditions. There were a few very good attempts though.

B7.3 Further Quantum Theory

- Problem 1: Part (a) was bookwork and uniformly answered well. Part (b)(i) was an exercise in raising and lowering operators for the harmonic oscillator. It could be done quite quickly with a bit of strategy, but many candidates spent significant time and space doing the manipulations inefficiently, often leading to mistakes. Part (b)(ii) required reasoning combining the algebraic properties of the harmonic oscillator with the theory of angular momentum. This was problematic for many candidates, and few collected many marks. Part (c) was a small elaboration on part (b), requiring the rule for addition of angular momentum and a small calculation similar to that used in considering spin-orbit coupling for the Hydrogen atom. This part was frequently left unanswered, though a small number of candidates made it most of the way through.

- Problem 2: This problem was the least frequently attempted by the candidates. Part (a) was bookwork, and almost all candidates carried it out without trouble. Part (b) was also almost book work, though the precise formulation may have been unfamiliar. Several candidates got full marks here, though some others didn't give a complete treatment. It was necessary to give the specialization of the general WKB expression to the case where the wave function was bounded at the origin. Parts (c) and (d) were quite similar to problems that were on a problem sheet, and required imposing the appropriate Bohr-Sommerfeld quantization rule and performing the relevant integrals. Candidates who made a real attempt at these parts largely were successful, though with some small errors arising. Part (e) required one to approximate the forbidden-region WKB wave function in the neighborhood of the origin; this was mostly not answered well.

- Problem 3: This problem was the most frequently attempted by the candidates. Part (a) was bookwork and usually answered very well, though a number of candidates lost a mark when not being sufficiently careful about the details of the argument. Part (b)(i) was a variational problem and required computing the expectation value of the Yukawa potential in the Hydrogen-like ground state. This could be done more quickly using the virial theorem, though it wasn't strictly necessary. A good number of candidates did the calculation of the Rayleigh quotient well, but fewer correctly explained the requirement for the existence for a bound state. Many had an issue with inequalities arising from not realizing that E_0 was negative. Part (b)(ii) caused a lot of problems. One had to expand the Yukawa potential as a power series in the parameter α and then use the various terms in the expansion to formulate a time-independent perturbation theory calculation. Especially important was that the $O(\alpha)$ term was a constant, and so could be treated exactly, with the first nontrivial perturbative term appearing at $O(\alpha^2)$. This meant that the *second-order* effect due to the leading nontrivial term term would contribute at $O(\alpha^4)$, allowing the α^3 term in the expansion of the potential to be treated in first order perturbation theory. Many candidates failed to put this all together, often computing fewer terms than were possible or not explaining the structure of the calculation adequately.

B8.1: Probability, Measure and Martingales

Given the open-book conditions, the more standard parts were naturally done accurately by most candidates, but some of the more novel parts were also well done by plenty of people (with the exception of the final part of Q3).

Q1: 1(a)(iii), although novel, requires only a small change to the standard proof of Kolmogorov's 0-1 Law, and most candidates had no problem with it (although a few didn't correctly read the definition of 1-dependence). 1(a)(iv) is a good test of basic probabilistic reasoning. The example in (b)(iii) is somewhat different to ones that had been seen in the lectures or example sheets; around a third of those attempting the question gave a more or less complete answer to this part.

Q2: 2(a)(iii) had appeared on an example sheet and didn't present much problem to most candidates. Just under a third of those attempting the question successfully found the variance in the final part, using the martingale $X_n^2 - 2n/3$.

Q3: The variance of marks on this question was fairly low. There were plenty of straightforward marks available at the beginning of the question. Part (b)(iv) is certainly challenging – I was hoping that some candidates would make good progress with it, given the hint, but none were successful.

B8.2: Continuous Martingales and Stochastic Calculus

Question 1 was attempted by almost all candidates, and was generally well done. Candidates lost marks principally for not being clear about the role of no-arbitrage in their arguments, or failing to explain why the PDE should hold for all $S > 0$. In part (b), many answers were not clear in their explanations of the role of Delta and Gamma in hedging. Part (d) was generally well done, but the comments on the sensitivity of the portfolios to changes in the price were not generally clear – many answers suggested a misunderstanding of the infinitesimal role of Delta (it only describes the behaviour for small changes in the price).

Question 2 was attempted by the majority of candidates. There was significant difficulty with part (a), with many answers failing to even mention the no-arbitrage price of the option, instead simply stating Feynman-Kac's statement of the relationship between the solution of the PDE and the expectation under $\mathbb{Q}$. Part (b) also proved difficult – many candidates first derived the density of S_T and then attempted to compute the expectation (rather than working with the closed form solution for S_T in terms of W_T and using the density of W_T, which is simpler). Some candidates also falsely assumed that the expectation of the product $1_{S_T>K}S_T^{\beta}$ is simply the product of the expectations of $1_{S_T>K}$ and S_T^{β}.

Part (c) was generally well done, but most answers did not recognise that the solution they had derived is only the price of the barrier option *assuming the barrier has not been hit*. Part (cii) also led to difficulty, with candidates assuming that the value of $S_{T_1}^{\beta}$ at time zero is simply S_0^{β} (which is not the case for $\beta \neq 1$).

Question 3 was not widely attempted. I suspect this is in part due to the absence of discrete time questions on recent past exams. Part (biii) caused difficulty, with very few answers pointing out that all payoffs can be hedged in a binomial market. Part (ciii) also was not well answered, as the payoff of the forward contract needs to be discussed.

Summary: Overall this exam was well done, particularly given the difficult circumstances arising from the sudden move to take-home examinations due to covid-19.

B8.3: Mathematical Models of Financial Derivatives

Question 1 was attempted by almost all candidates, and was generally well done. Candidates lost marks principally for not being clear about the role of no-arbitrage in their arguments, or failing to explain why the PDE should hold for all $S > 0$. In part (b), many answers were not clear in their explanations of the role of Delta and Gamma in hedging. Part (d) was generally well done, but the comments on the sensitivity of the portfolios to changes in the price were not generally clear – many answers suggested a misunderstanding of the infinitesimal role of Delta (it only describes the behaviour for small changes in the price).

Question 2 was attempted by the majority of candidates. There was significant difficulty with part (a), with many answers failing to even mention the no-arbitrage price of the option, instead simply stating Feynman-Kac's statement of the relationship between the solution of the PDE and the expectation under $\mathbb{Q}$. Part (b) also proved difficult – many candidates first derived the density of S_T and then attempted to compute the expectation (rather than working with the closed form solution for S_T in terms of W_T and using the density of W_T, which is simpler). Some candidates also falsely assumed that the expectation of the product $1_{S_T>K}S_T^\beta$ is simply the product of the expectations of $1_{S_T>K}$ and S_T^β.

Part (c) was generally well done, but most answers did not recognise that the solution they had derived is only the price of the barrier option *assuming the barrier has not been hit*. Part (cii) also led to difficulty, with candidates assuming that the value of $S_{T_1}^\beta$ at time zero is simply S_0^β (which is not the case for $\beta \neq 1$).

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Summary: Overall this exam was well done, particularly given the difficult circumstances arising from the sudden move to take-home examinations due to covid-19.

B8.4: Information Theory

Question 3 was the most popular with nearly all candidates attempting it. Question 1 and Question 2 were approximately equally popular.

Question 1: For part a) the candidates got almost all marks, omitting the "iff"-part of Gibbs' inequality was the main reason for point deductions. In part b) a few candidates successfully adapted the concise approach seen for similar problems on the problem sheets and in the lectures. However, a large number of candidates instead tried to work with Lagrange multipliers. They typically stopped after re-deriving the pmf given in the problem without showing (global) maximality and many failed to accommodate for the moment constraint being an inequality. In part c) most candidates made some progress in the computations, but only a few managed to keep an overview and find a suitable factorisation.

Question 2: In part a) some marks were lost due to random variables taking negative values and various imprecisions. On part b) almost all candidates scored high. Similarly, for part ci) where again some marks were lost for not justifying the extremum being a (global) maximum. In part cii) some candidates struggled with finding an explicit formula for the error-probability of the resulting BSC.

Question 3: Most candidates successfully completed the book work, part a), and succeeded in constructing the Huffman codes, part b). Many candidates also made good progress on part c) with dealing with odd length codewords in a way that leads to a uniquely decodable code. A small number of candidates proved the stronger boung ciii) directly and deduced the upper bound in cii) from it.

B8.5: Graph Theory

Overall this paper was perhaps on the hard side, although one candidate obtained full marks. This script was exceptional, going significantly further than required to obtain 100%.

Question 1 was attempted by almost all candidates. It contains the most bookwork, and ought to be straightforward. (a) was mostly well done, though quite a few candidates gave complicated answers to (ii) rather than a simple modification of the argument in lectures for (i). (b)(i) is bookwork. Many candidates copied out the proof from notes of a more general result and then applied it. This is fine, but it can be shortened a bit in context. The example for (b)(ii) is very simple, but not that many found it. Part (c) distinguished well. The idea is to use Menger's Theorem via the Fan Lemma, to extend a cycle that is shorter, in a similar manner to on a problem sheet (in a different context). There were many incorrect proofs given here. (E.g., if you have one x-y path, Menger does not let you find another one independent of it.) There is a simple example (with some variants) for the last part. It proved (as hoped!) moderately tricky to find.

Question 2 was mostly well done, though part (b) was disappointing. Many arguments for (i) involved changing the drawing of H (which is not allowed), or, if correct, would have shown that any two vertices in different components can be joined, which is not true. (Consider a triangle and two isolated vertices, one inside and one outside.) For (ii), some lengthy arguments were given; using (i) gives a very short argument. (c)(i) was mostly well done, with some small missing details (e.g., noting that unless G is a forest, the boundary of every face must contain a cycle). Examples for (c)(iii) seemed to be easy to find, those for (iv) not so much. For (v) just mentioning the Four Colour Theorem is enough, although it is possible to give a proof by modifing the argument in lectures for the Five Colour Theorem.

Question 3 was least popular, and was not well done. It seemed that not many students absorbed even the rough ideas of the last part of the course, even though this is just as important as the earlier parts. (a) is, as mentioned in the notes, a simple *modification* of the proof for the bipartite case in notes. No credit was given for copying out the proof from notes – candidates were expected to show understanding by carrying out the modification. The key idea in part (b) is to use the corollary of Erdős–Stone, which says that to first order, the chromatic number determines the extremal number. Then one has to argue by hand to compare the bipartite graphs, with a bit of work needed for the strict inequalities. For (c) one can start by using Hall's Theorem.

BO1.1: History of Mathematics

Both the extended coursework essays and the exam scripts were blind double-marked. The marks for essays and exam were reconciled separately. The two carry equal weight when determining a candidate's final mark. The first half of the exam paper (Section A) consists of six extracts from historical mathematical texts, from which candidates must choose two on which to comment; the second half (Section B) gives candidates a choice of three essay topics, from which they must choose one. The Section B essay accounts for 50% of the overall exam mark; the answers to each of the Section A questions count for 25%.

Throughout the course, candidates were invited to analyse historical mathematical materials from the points of view of their 'context', 'content', and 'significance', and these are the three aspects that candidates are asked to consider when looking at the extracts provided in Section A of the exam paper. Indeed, most candidates chose to use these as subheadings within their answer — this is entirely acceptable, although several candidates had a tendency to place details under the wrong headings. Another common pitfall in the handling of the extracts questions was that candidates failed to address the content of the given extract closely enough, giving instead a much more general account of the associated topic.

The Section A questions 1–6 were tackled by 2, 2, 5, 4, 4, and 3 candidates, respectively. Questions 1, 2 and 6 were the more difficult questions: the first because it related to a topic that was touched upon only briefly within one lecture, the second because it concerned a rather involved and largely numerical manuscript source (though this had been seen), and the last because it required the interpretation of an interpretation, namely an editorial introduction to Cantor's work, rather than Cantor's own words. Curiously, the extract in question 6 was misidentified by some candidates as being a definition of completeness. The relative popularity of questions 3 and 4 is explained by their relating to core ideas from the lecture course concerning the development of analysis. They were generally well done, although an important point missed by some candidates who attempted question 4 was that Lagrange's study of remainder terms for Taylor series played a role in establishing the study of inequalities of finite quantities as the basis for analysis. Some answers to question 5 seemed to suggest anachronistically that (abstract) group theory was an established field to which Cauchy contributed, and failed to emphasise that Cauchy was dealing not with abstract groups, but solely with groups of permutations.

In Section B of the exam paper, questions 7–9 were attempted by 5, 1, and 4 candidates, respectively. The unpopularity of question 8 is rather surprising, given that (like questions 3 and 4) it related to a core theme of the lecture course. Candidates had encountered plenty of examples of Euler's work, both in reading and in lectures, which explains why question 7 was the most popular question here. It was generally well done, though some answers tended towards the superficial. Question 9 was intended as a harder question, since possible starting points are less clear than for the other questions, and so it is a little surprising that so many candidates chose to attempt it. Answers to this question were interesting, and brought up points not previously considered by the setter, but the assessors were not quite convinced by the argument put forward by more than one candidate that linked abstraction solely to pure mathematics.

The extended essays featured good use of primary sources, with some candidates reproducing appropriate diagrams from those sources. Indeed, other candidates lost marks because their verbal descriptions of geometrical representations of complex numbers were difficult to follow without diagrams — clearer explanations would have been afforded by the inclusion of well-chosen diagrams that were clearly and explicitly linked to the text. More generally, the amount of mathematical detail included by candidates was quite varied; those who included little should not have been afraid to give more. Some essays were penalised because they promised things in their introductions that did not then feature in the essay, or otherwise had conclusions that did not match up with the introductions.

Statistics Options

Reports of the following courses may be found in the Mathematics & Statistics Examiners' Report.

SB1.1/1.2: Applied and Computational Statistics

SB2.1: Foundations of Statistical Inference

SB2.2: Statistical Machine Learning

SB3.1: Applied Probability

SB3.2: Statistical Lifetime Models

SB4: Actuarial Science

Computer Science Options

Reports on the following courses may be found in the Mathematics & Computer Science Examiners' Reports.

CS3a: Lambda Calculus & Types

CS4b: Computational Complexity

Philosophy Options

The report on the following courses may be found in the Philosophy Examiners' Report.

102: Knowledge and Reality

127: Philosophical Logic

129: Early Modern Philosophy

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